Scale invariance

Scale invariance

An object is invariant when a transformation leaves it unchanged. A circle is invariant by rotation, since whether a circle is rotated 10° or 60°, the same circle is still obtained. An object is scale-invariant when it remains unchanged by the action of a direct or reverse (retraction) dilatation. Scale invariance indicates that no scale, whether temporal or spatial, characterises the object, so that each part resembles the whole. As each part is like the whole, the phenomenon is said to be self-similar. This amounts to saying that all scales contribute equally to the state of the object observed [Laguës & Lesne, 2008].

Scale invariance was initially observed in mathematical figures and in certain plant forms. The Apollonian gasket or Apollonian net is probably the first self-similar construction. Later, Albrecht Dürer assembled small pentagons starting from the five sides of a larger pentagon, a premier in the area of central place models. In science, however, the rule was rather to associate a scale, in terms of time or space, with each phenomenon or object. To understand an object on a given scale, scientists took account of phenomena on a lower level, and then on a higher level. This hierarchical method [Sheppard & McMaster, 2004] demonstrated the interest of multi-scale approaches. But while all geographical systems are multiscalar, they are not all scale-invariant.

Scale invariance gained ground with the work by L-F Richardson, and later B. Mandelbrot [1982]. Scale invariance, found in the forms known as fractals, also applies to frequential models, to temporal series or to processes. Scale invariance corresponds to a power law.

However, when processing observed data, scientists note that the condition of self-similarity is very restrictive [Queiros-Condé et al, 2015]. First of all, scale invariance can only be verified between two levels. Second, numerous anomalies arise on log-log graphs. Sometimes two or three slopes appear. This provides a semi-fractal. More often, the slope alters in continuous manner, in particular at the extremities. This produces a parabolic fractal where the fractal dimension changes according to the range of scales considered. In a two-dimensional space, a thermal field or a satellite photograph of a city, self-similarity can vary with the direction. In this case the fractal exhibits self-affinity.

Regarding multi-fractals, these characterise objects where several imbricated fractal dimensions are to be taken into account [Seuront, 2010]. For instance the built-up zones of a city shown on a satellite image in 256 levels of grey. It is possible to distinguish the other components (green spaces, roads etc.) and thus to create a binary image and calculate it fractal dimensions ([Frankhauser, 1994]. But it is also possible to construct a different binary image, contrasting one-storey buildings from the other spaces, and to obtain a new fractal dimension. By repeating this process, if all the fractal dimensions calculated are equal, the city is mono-fractal. If the fractal dimensions differ, the city is multi-fractal. Thus a multi-fractal object locates and quantifies the singularities or irregularities on all scales. The multi-fractal approach thus reconciles the **nomothetic** approach and **idiographic** analysis.

Scale invariance, and above all any departure from this principle, can be seen in all geographical phenomena – relief, climate zones, hydrographic networks, cities, urban networks and communication networks, from underground trains to Internet. Since scale invariance is also verified in most disciplines, including artistic disciplines, a whole range of tools is on offer. Among the most widespread are the tools constructed on the basis of a breakdown into wavelets or entropic approaches [Dauphiné, 2012]. Finally, to explain scale invariance, scientists also call on the **theories of complexity**.

Bibliographie