Fractal

(term proposed by B. Mandelbrot)

In mathematics, a fractal is a geometrical object such that the same properties are showing at different scales. There is thus a hierarchically organised structure, which follows a well-defined hyperbolic law, either determinist (regular fractal) or stochastic (random fractal):

$$n(e) = e^{-D}$$

where e is the measure interval(variable parameter characterising the scale of observation) and n the number of elements appearing at that scale. D is the fractal dimension, a parameter that characterises distribution of quantities (masses, areas, lengths) through different observation scales: it describes the internal homothety of the observed structure. The value of this D dimension is empirically determined on basis of the slope of the hyperbolic relation adjusting quantities and measure interval. For a surface inside a volume, D lies between 2 and 3, for a curve on a surface, between 1 and 2.

The fractal dimension is used in geography in order to generalise delineation of coasts on maps, or to test existence of a hierarchical organisation in various systems (urban morphology, distribution of population, central places, transport networks...). If the structure is fractal, the quantities (e.g. built area, length of a network) are functions with a negative power of an interval of measure (width of the cell of an analysis grid, distance to a centre, width of a lane...).

Bibliographie